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Comments on D-Brane and $SO(2N)$ Enhanced Symmetry in Type II String

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Abstract

We propose a configuration of D-branes welded by analogous orbifold operation to be responsible for the enhancement of $SO(2N)$ gauge symmetry in type II string compactified on the D_n-type singular limit of K3. Evidences are discussed from the D_n-type ALE and D-manifold point of view. A subtlety regarding the ability of seeing the enhanced $SO(2N)$ gauge symmetry perturbatively is briefly addressed.

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It is well known that when N parallel D-branes coincide, the N copies of the $U(1)$ gauge field get enhanced to the non-abelian $U(N)$ gauge theory on the worldvolume[1]. This has been employed in the context of type II string compactified on the singular limit of K3 or C-Y to explain enhancement of A_n -type gauge symmetry when there are 2- or 3-cyles on the manifold shrinking to zero or to a curve [2][3][4][5]. The work of [2] also interprets the appearance of the $U(N) \times U(M)$ gauge group as the result of two sets of D-branes intersecting on their common transverse coordinates. As explained in [6] and [7], enhanced gauge symmetries can appear only when the underlying conformal field theories become singular. Indeed it was suggested in [7] and further demonstrated in [8] that at the moduli space where the type IIA string compactified on the ALE manifold has A_n type enhanced gauge symmetry, the singular conformal field theory is such that it develops a semi-infinite tube similar to a symmetric fivebrane [9] which admits exact (4,4) superconformal description.

Little is still understood about the other examples of enhanced gauge symmetry like D_n and $E_{6,7,8}$ series which are believed to appear when the (transverse) singularities are the corresponding type of ALE spaces. The work of [8] has suggested a general procedure to study the ADE type singularity in the local models of K3 compactification. It is essential that by performing discrete modding of the local WZW coset at specific levels one obtains, through analog of the stringy cosmic string construction [10], exact CFT of the symmetric fivebrane with H-charge. Although [8] only worked out details for the A_n series, generalizations to other types of ALE singular space are possible. We will deal with some delicacies for the D_n series in the following. Similar arguments involving T- and S-duality [2] lead us to the D-string on D-manifold (a manifold with a set of D-branes on it) in the type IIB setting, where the enhanced gauge symmetry gets explained in terms of the Chan-Paton factors of the open strings stretched between the D-branes. In this letter we will propose a non-compact D-manifold which consists of D-branes welded by an analog of Z_2 orbifold operation dictated specifically by the D_n -type ALE space. Consideration of the compact D-manifold has led to Vafa's F-theory [11]. Our final picture seems to be related to the dynamical D-branes in the type I string compactified on the Z_2 orbifold limit of K3 [12][13], though the differences are apparent, because we don't do orientifolding which would bring type IIB to type I, and the ALE spaces are non-compact thus no tadpole concellation and no 9-branes.

That the D-manifold with two sets of n D5-branes related by a Z_2 leads to $SO(2n)$ enhanced gauge group through Chan-Paton factors can be roughly seen as follows. Suppose placing n parallel D5-branes at the point $y^i = a^i, i = 1, ..., n$ in the tranverse directions, and another set of n parallel D5-branes at the point $y^i = b^i, i = 1, ..., n$. When these D5-branes coincide $a^i = a, b^i = b$, one gets for each sets separately an enhanced group $U(n)$. Now if the two sets of branes are such that their transverse coordinates are related by a Z_2 transformation $a^i = -b^i$, and the $U(1)$ gauge fields associated with individual D5-branes in different set are also identified by the induced Z_2 action, then, one gets the enhanced $SO(2n)$ gauge symmetry with massless adjoint

vector multiplets coming from open string states that would have given rise to $U(n) \times U(n)$ adjoint had the two sets of branes been arranged to intersect. Note that to obtain the above spectrum, it is crucial that the open string states stretching from D5-branes in one set to those in another won't be counted (this is the major distinction from the type I orientifold), as their masses are proportional to $T|a^i - b^j| \sim 2T|a|$ which is not vanishing unless $a, b = 0$. In the later case the configuration is actually equivalent to $2n$ parallel D5-branes giving rise to $U(2n)$. It is a degeneration of the $SO(2n)$ configuration and it is believed that there are obstructions for the configuration to decay into $a, b = 0$. So we will treat the generic case that $a, b \neq 0$ here. It should be mentioned that D-branes on the ALE space of type D_n must be more severely constrained than they are in the orbifold limits of K3, since the D_n -type ALE space is essentially a non-abelian orbifold. The relation $a^i = -b^i$ above should not be taken as the Z_2 identification of the underlying orbifold, rather it is a result of nontrivial dynamics of the D-branes moving on the ALE space. The two sets of D-branes are being brought to the positions $a^i = -b^i$, but there are no Z_2 identifications of the coordinates of the ALE space.

To give evidence supporting the above picture, we will study the singular CFT of the D_n -type ALE space and see if D_n -type D-manifold arise naturally. It is worthwhile repeating some logic steps outlined in [2][8].

Guided by the conifold in Calabi-Yau [14], the work of [8] advocates the role of 2d massless black holes in the description of K3 and Calabi-Yau singularities, generalizing the previous partial results in [7]. Starting from type IIA string compactified on singular limit of K3, one models the local geometry of the singular K3 by the kind of Landau-Ginzburg (orbifold) models which, by analogous arguments of the $c = 1$ string at self-dual radius, are of the form of the (Gepner) product of two coset models $SL(2)/U(1) \times SU(2)/U(1)$ with relevant affine levels dictated by the ADE types of the minimal model modular invariants and the requirement that the complete models have ($N = 2$) central charge $\hat{c} = 2$. The non-compact Kazama-Suzuki model $SL(2)/U(1)$ at the critical point is inspired directly from the 2d Euclidean black hole which admits a semi-infinte cigar as its bosonic geometry with a linear dilaton in the direction of the length, and in addition, a circular scalar with radius associated with the affine level. By performing discrete group modding of the L-G superpotentials one obtains for the case of A_{n-1} and $G = Z_n$, a system equivalent to the capped version of the symmetric fivebranes with H-charge n. Implicit in the construction is a T-duality which exchanges the A- and B-model descriptions and consequently type IIA on ALE becomes type IIB on the symmetric fivebranes. Now type IIB in ten dimensions admits $SL(2, Z)$ self-duality and therefore applying the S-duality to the type IIB string on (NS-NS) fivebranes leads us to the realm of D-string (the S-dual of the type IIB string) on the D-5-branes (the S-dual of the NS-NS fivebranes). The authors of [2] went on further to construct several examples of D-manifold which consists of D-brane skeletons with open strings connecting them. When these D-branes become coinciding, the massless states which are now interpreted as excitations of the D-string appear and give rise to vector as well as hypermulitplets

of the extended gauge theory on the worldvolume of the D-5-branes.

We are interested here in seeing similar things happening in the case of D_n series. At first glance, it seems difficult to conceive the appearance of the fivebrane CFT by applying directly the discrete group modding to the L-G model of the D_n type. Actually the case of D_n series corresponds to a non-Abelian orbifold which seems less understood in the previous studies. At best we could hope for an *orbifold* of the fivebrane CFT. This is not really bad since we don't expect parallelly aligned *bona fide* fivebranes to give rise to the D_n type enhanced symmetry. Keeping in mind the anticipation of the D-brane welding stated a while before, we are going to extrapolate a Z_2 action from the non-Abelian orbifold which should act on the fivebranes (and subsequently on D-5-branes) as anticipated.

For the type IIA string on $K3 \times \mathbf{R}^6$ with the D_n type singularity of K3

$$D_n: \quad x^{n-1} + xy^2 + z^2 = 0, \quad n \geq 4, \tag{1}$$

locally the Calabi-Yau geometry can be modelled by the Landau-Ginzburg superpotential

$$W_{D_n} = -\mu w^{-2(n-1)} + x^{n-1} + xy^2 + z^2. \tag{2}$$

It can be verified that $W_{D_n} = 0$ satifies the Calabi-Yau condition with $\hat{c} = 2$. The different parts of the terms in (2) (x, y, z) and (w) are associated with the $N = 2$ superconformal minimal model coset $SU(2)_{2(n-2)}/U(1)$ of type D_n modular invariants and the non-compact Kazama-Suzuki coset $SL(2, R)_{2n}/U(1)$ at level $2n$, respectively. As in the A_{n-1} case, one is obliged to perform modding by a discrete group G of the product of cosets. Unlike the case of A_{n-1}, here the discrete group G happens to be the rank $(n-2)$ dihedral group, $\mathcal{D}_{n-2}$ as it is the symmetry of polynomial W_{D_n} in eq(2). Modding out the CFT by general (solvable) discrete groups has been studied in [15], and we will use some of their results in the following.[1]

The group $\mathcal{D}_{n-2}$ can be taken as semi-direct product of Z_2 and Z_{n-2} and is defined by two generators θ, τ satisfying relations

$$\tau^2 = \theta^{n-2} = 1; \quad \tau^{-1}\theta\tau = \theta^{-1}. \tag{3}$$

Generator θ corresponds to rotation of the $(n-2)$-sided polygon through an angle $2\pi/(n-2)$, while generator τ a rotation of π about an axis of symmetry. The stabilizer subgroups are $N_1 = \mathcal{D}_{n-2}$, $N_\tau = Z_2$, $N_\theta = Z_{n-2}$. There are two or four 1-dimensional irreducible representations and $\frac{1}{2}(n-3)$ or $\frac{1}{2}(n-4)$ 2-dimensional irreducible representations according to whether n is odd or even. The 1-dimensional ones act simply as

[1] Near the completion of this work, we became aware of the references [16, 17] where non-Abelian orbifolds of CP^1 and CP^2 by dihedral groups have been studied in connection with cohomology rings of the σ-model target spaces.

sign-changes while the 2-dimensional ones are generically of the form

$$\sigma_i(\tau) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_i(\theta) = \begin{pmatrix} \alpha^i & o \\ 0 & \alpha^{-i} \end{pmatrix}, \quad i = 1, ..., l, \tag{4}$$

where α is the $(n-2)$-th root of unity, and l is the number of the 2-dimensional representations in either cases. The two dimensional representation (4) is helpful when we notice that the WZW models of $SL(2,R)$ and $SU(2)$ both admit natural actions of $\mathcal{D}_{n-2} \subset SU(2)$. The detailed spectrum and operator content of this (non-abelian) orbifold conformal field theory can be worked out, according to [15], at least for the cases of solvable groups. We are not going to bring the burden of a complete analysis of various sectors of chiral rings into the present work, but only note that in the fixed-point-free[2] case, the linear (i.e., not projective) representations in the untwisted sector give rise to operators which are in 1-1 correspondence with the nodes of the extended Dynkin diagram. We ignore the twisted sectors and the related fusion rules since we are not concerned with solving the orbifold CFT completely.

An essential point of the non-Abelian orbifolds studied in [15] is that since the group $\mathcal{D}_{n-2}$ is (super)solvable, which means that there is an exact sequence

$$0 \to Z_2 \to \mathcal{D}_{n-2} \to Z_{n-2} \to 0 \tag{5}$$

such that $\mathcal{D}_{n-2}$ is the extension of Z_{n-2} by Z_2, one can perform the orbifolding by $\mathcal{D}_{n-2}$ through a sequence of Abelian orbifolds by Z_{n-2} and Z_2. If one performs first modding out by Z_{n-2} and subsequently performs modding by Z_2, the action of the Z_2 is to identify the operators obtained by conjugating the representations in each sector of the twisted Hilbert space. However, we note that by modding out Z_{n-2} we always get trivial bosonic part of the WZW $SU(2)$ model with its free scalar field at radius 1. Thus it seems impossible to see the fivebrane CFT that is a WZW $SU(2)$ model at nontrivial level and a Feigin-Fuchs field. Examining the affine levels of both factors of the product coset, one sees that if one scales the momentum lattice of the free bosons by a factor of $\frac{1}{\sqrt{2}}$, then modding out by a cyclic group Z_{n-1} would result in a WZW model $SU(2)$ at level $n-2$ [3] together with a Feigin-Fuchs field which should correspond to a version of symmetric fivebrane with the H-charge n.

This seems to be what we want. But how could we modd by Z_{n-1} for a $\mathcal{D}_{n-2}$ system? Z_{n-1} is the group of monodromies of the D_n singularity (see for example [18]), modulo

[2]Fixed-point in the sense of fusion algebra, not to be confused with the geometric sense of ALE.

[3]The effect of modding out Z_{n-1} is to recombine two bosons from the product WZW cosets into

$$\tilde{X}^1 = p_1 X^1 - p_2 X^2$$

$$\tilde{X}^2 = \sqrt{(n-2)} p_1 X^1 + X^2$$

with radii $\tilde{R}_1 = 1, \tilde{R}_2 = \sqrt{2(n-2)}$.

the Z_2 center (i.e. the full-fledged monodromy group is $Z_{2(n-1)}$ instead of Z_{n-1}). By modding out by Z_{n-1} we did nothing wrong except trivialized the monodromy around the y coordinate in eq.(1). Incidently, letting $y = 0$, one recovers, up to quadratic perturbation, an A_{n-2}-type singularity, hence orbifolding by Z_{n-1} (and letting $w' = w^2$ in eq.(2))gives CFT of fivebranes of H-charge $n-1$, as in [8]. Compare to the result in the last paragraph, we see an extra fivebrane has been trapped in the trivialization of the y-monodromy.

Another point in the above argument is the effect of rescaling the momentum of the free boson by a factor of $\frac{1}{\sqrt{2}}$. What does it do to the fivebrane picture? Note that this has effectively changed the Kähler moduli parameter. If we identify the circle as a homology circle in the torus fiber of the ALE space (after modelling it by analogous stringy cosmic string construction), this rescaling corresponds to modifying the B field period by one half, $B \to B + \pi$. The H-charge n which we got before actually consists of contributions of $2n$ symmetric fivebranes each carrying H-charge $\frac{1}{2}$.

Now it is clear how the remaining orbifold by Z_2 act on the $2n$ fivebranes. There are $(n-1)$ pairs of them identified by the Z_2 as inner automorphism of the A_{n-2} extended Dynkin diagram; the other two are exchanged via the outer automorphism Z_2 which effectively splits off the fivebrane trapped when trivializing the y-monodromy. It would be a challenge to verify this explicitly by studying the inverse question of orbifolding the CFT of the $2n$ symmetric fivebranes and deriving the D_n-type ALE geometry as an object whose affine coordinate ring coincides with the truncated chiral ring of the orbifold CFT.

Having exploited the relevant features of the (non-abelian) orbifold, we would like to further pursue the idea that D-string on D-manifold of the D_n-type indeed gives rise to enhanced gauge symmetry $SO(2n)$. Now we have a picture of type IIB on the orbifold of the fivebranes with H-charge n, according to [2], the S-duality of the 10 dimensional type IIB string theory leads to the dual picture of D-string on a manifold with D-branes. There is a subtlety though, that is when we transform the B field by the $SL(2, Z)$, we would still get n D-5-branes. The point is that the group $SL(2, Z)$ has a mod 2 congruence subgroup $\Gamma_0(2) \subset SL(2, Z)$, which upon conjugation by the element $\delta = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \in SL(2, Z)$ dictates the kind of transformation of the B field we found earlier. Thus if we use $SL(2, Z)/\delta \circ \Gamma_0(2)$ as the Z_2 part of the S-duality group of the type IIB string theory, then obviously the NS-NS fivebranes of charge n get transformed into $2n$ D-5-branes.

To see the enhanced gauge symmetry of type $SO(2n)$ in type IIA string compactified on D_n -type sigular limit of K3, we are looking for the relevant D-manifold on which the dual of the type IIB string get excitations of the form of the corresponding gauge multiplets. It is straightforward to generalize the arguments of [2] to the D_n -type D-manifold, and find out the massless spectrum of the $N = 2$ $SO(2n)$ gauge theory.

Let us see how the pure gauge content arise simply by looking at what the D-manifold looks like. For the case of A_{n-1} series, the D-manifold coincides with the dual of the corresponding extended Dynkin diagram [4] (see fig. 1 of the paper [2]), where an extra vertex is added which is expressable in terms of the $(n-1)$ simple roots of $SU(n)$

$$r_0 = -\sum_{i=1}^{n-1} r_i$$

where $r_i = \mu_i - \mu_{i+1},\ i = 1, ...n-1$ are the simple roots. Note that when $\mu_i = \mu_{i+1}$, signalling vanishing of the the $(n-1)$ 2-cylces, the extra cycle vanishes trivially (as dictated by homology relation), though it is interpreted as appearance of massless hypermultiplets in the adjoint. The D-manifold dual to the extended Dynkin diagram can be viewed as representing n D-branes located at different positions with links connecting them being identified as masses of the stretched string states, which go to zero when the D-branes coincide. The extended Dynkin diagram of the D_n type consists of $n+1$ vertices, the $(n+1)^{th}$ vertex is defined as

$$r_0 = -\sum_{i=1}^{n} a_i r_i \tag{6}$$

where r_i denote the simple roots of $SO(2n)$, $\mu_1 - \mu_2, ..., \mu_{n-1} - \mu_n, \mu_{n-1} + \mu_n$, and a_i are the dimensions of the irreducible representations of the groups $\mathcal{D}_{n-2}$. We recall that the number of two dimensional characters of $\mathcal{D}_{n-2}$ is $\frac{n-4}{2}$ or $\frac{n-3}{2}$ depending on whether n is even or odd. we note that vanishing of the n 2-cycles must be accompanied with a Z_2 symmetry of reflection on the μ_n plane. This is part of the Z_2 symmetry belonging to the Weyl group of $SO(2n)$ root system. Now it is clear where to put our $2n$ D-branes on the (resolved) ALE space $X_{ALE} = \mathbf{C}^2/\mathcal{D}_{n-2}$. We identify the root lattice of the simply-laced group with the second homology group $H_2(X_{ALE}, Z)$ of the (resolved) ALE space X_{ALE}, the vertices of the Dynkin diagram can be taken as the vanishing 2-cycles whose areas are proportional to the distance of the nearby D-branes $\mu_i - \mu_{i+1}$. We imbed the root lattice of the two copies of the A_{n-2} system into that of the D_n system. According to the previous discussions, the $2(n-1)$ D-branes are identified pairwise by the inner automorphism Z_2 of the A_{n-2} system as (pairwise) permutations of the vectors $\mu_i, i = 1, ..., n-1$. There are also two D-branes which are exchanged by the outer automorphism (of the A_{n-1} system), here represented by the Weyl reflection on the vector μ_n. The dual picture of this D_n -type D-skeletons is depicted as the extended Dynkin diagram in Fig. 1 below (with $n = 6$)

[4]The extended Dynkin diagrams of the simply laced Lie algebras with specific categories are named quivers[19]. These quiver diagrams contain more information than the extended Dynkin diagrams and are quite useful in studying ALE instantons via ADHM construction. See [20][21] for the discussion of this from mathematical and physical points of view.

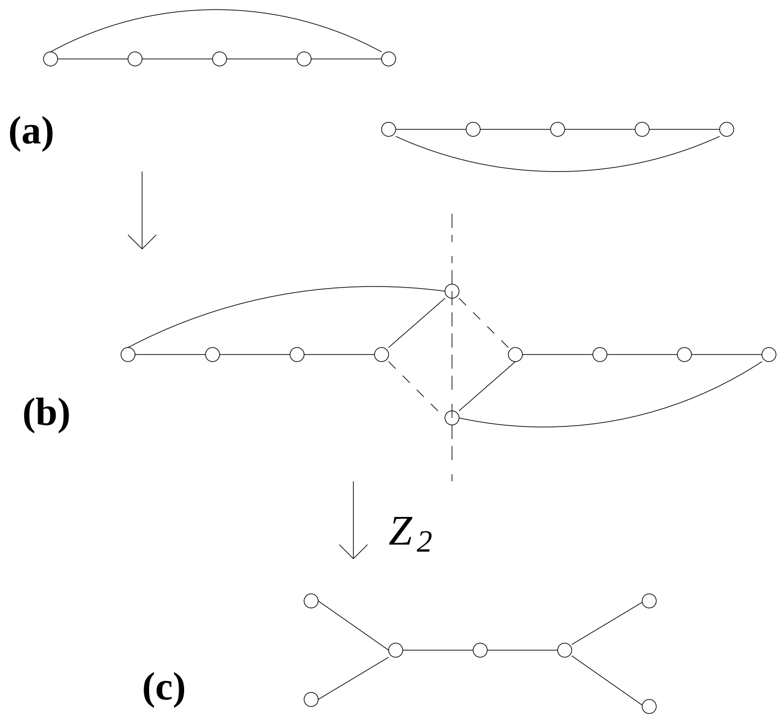

fig 1. A sequence of steps for obtaining D_n extended Dynkin diagram from two A_{n-2}'s.

In Fig.1b, one sees the hidden D-brane between the $(n-2)^{th}$ vertex and another vertex which is the image of the $(n-1)^{th}$ vertex under outer automorphism.

Before closing, we mention that there is a subtlety in the enhanced gaueg symmetry of the D_n series. Namely, one might ask whether this kind of enhanced gauge symmetry can be seen perturbatively in the heterotic duals or is it intrinsically related to some non-perturbative effects like small instantons in the heterotic $SO(32)$ theory [22]. There is indication that the vertex with three links ending on it should correspond to the 2-cycle whose vanishing can not be understood perturbatively [5]. In our case an outer automorphism acts nontrivially on this vertex, it must be checked whether this action is compatible with (possible) monodromy actions when fibering the D_n ALE space over a base. This and other questions related to string-string duality are certainly worth pursuing further.

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References

[1] E. Witten, Bound states of strings and p-branes, hep-th/9510135, Nucl. Phys. **B** (1996)

[2] M. Bershadsky, C. Vafa and V. Sadov, D-strings on D-manifolds, hep-th/9510225, Nucl. Phys.**B** (1996)

[3] S. Katz, D.R. Morrison and M.R. Plesser, Enhanced gauge symmetry in type II string theory, hep-th/9601108,

[4] P. Berglund, S. Katz, A. Klemm and P. Mayr, New Higgs transitions between dual N=2 string models, hep-th/9605154.

[5] P.S. Aspinwall, Enhanced gauge symmetry and Calabi-Yau threefolds, hep-th/9511171.

[6] P.S. Aspinwall, Enhanced gauge symmetries and K3 surfaces, hep-th/ 9507012, Phys. Lett. **357B**(1995)329.

[7] E. Witten, Some comments on string dynamics, hep-th/9507121.

[8] H. Ooguri and C. Vafa, Two-dimensional black hole and singularities of CY manifolds, hep-th/9511164, Nucl. Phys. **B463**(1996)55.

[9] C.G. Callan, J.A. Harvey and A. Strominger, Worldsheet approach to heterotic instantons and solitons, Nucl. Phys. **B359**(1991)635; Worldbrane actions for string solitons, Nucl. Phys. **B367**(1991)60.

[10] B.R. Greene, A. Shapere, C. Vafa and S.T. Yau, Stringy cosmic strings, Nucl. Phys. **B337**(1990)1.

[11] C. Vafa, Evidences of F-theory, hep-th/9602022.

[12] E. G. Gimon and J. Polchinski, Consistency conditions for orientifolds and D-manifolds, hep-th/9601038

[13] E.G. Gimon and C.V. Johnson, K3 orientifolds, hep-th/9604129; M. Berkooz and R. Leigh, A D=4 N=1 orbifold of type I string, hep-th/9605049; A. Dabholkar and J. Park, An orientifold of type IIB theory on K3, hep-th/9602030; A. Dabholkar and J. Park, Strings on orientifolds, hep-th/9604178.

[14] A. Strominger, Nucl. Phys. **B451**(1995)85.

[15] R. Dijkgraaf, C. Vafa, E. Verlinde and H. Verlinde, Commun. Math. Phys. **123**(1989)485.

[16] E. Zaslow, Topological orbifold models and quantum cohomology rings, Commun. Math. Phys. **156**(1993)301;

[17] E. Zaslow, Dynkin diagrams and CP^1 orbifolds, Nucl. Phys. **B415**(1994)155.

[18] E. Brieskorn, Math. Ann. **166**(1966)76.

[19] H. Nakajima, Duke Math. Journ. **76**(1994)365.

[20] P.B. Kronheimer and H. Nakajima, Math. Ann. **288**(1990)263.

[21] M.R. Douglas and G. More, D-branes, quivers, and ALE instantons, hep-th/9603167.

[22] E. Witten, Small instantons in string theory, Nucl. Phys. **B460**(1996)541